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General fractal-discrete scheme for high-frequency lung sound production

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A general scheme is proposed to explain the observed spectral properties of high-frequency human respiratory sounds in terms of the interaction between the respiratory flux and a bronchial tree of fractal properties. The air flux is treated as composed of discrete decoupled elements while the tree is assumed to have a Cantor-based geometry. According to this model, the affine behavior often observed in the high-frequency (log-log) spectral range is a direct consequence of the fractal geometry of the bronchial tree in both qualitative and quantitative aspects. This strongly indicates that the dynamics underlying the high-frequency sound generation must have at most nondominant couplings between the relevant fluid components.

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I. INTRODUCTION

In human inspiration and expiration curses, the generated respiratory sounds have their origin in the interaction of the air flux with the bronchial tree boundary. Although these sounds are frequently used by physicists as a first resource for pathology detection in the human respiratory system, the specific mechanisms of air flux to sound conversion are not completely understood yet. In fact, the complete understanding of respiratory sounds production should come with the solution of compressive fluid equations under a complex fractal-like set of boundary conditions, which represents a remarkable task.

The spectral characteristics of normal subject breath sound as determined by experiment are reported by the authors of Refs. [1,2]. Usually, for normal subjects the observed sound spectrum is divided into a low and high frequency region, respectively. The first typically ranges from approximately 75 to about 160 Hz while the subsequent range is limited by the maximal frequency reliably detectable at about 1000 Hz. In the present paper we focus especially on the latter spectral region, henceforth referred to as the *high-frequency region*. It is notable that this spectral range fits with considerable accuracy an affine feature in the log-log frequency-amplitude plane, suggesting a self-similar mechanism for sound generation. Figure 3 illustrates the typical shape of the amplitude spectrum.

Fractal geometry has been a useful guide for understanding many natural patterns since it seems to be a common optimization solution used by Nature. The bronchial tree, here denoted by $\mathcal{T}$, is one of the many examples found in the human physiology where a fractal geometry is verified (see Refs. [3–6] among others). In fact, $\mathcal{T}$ is composed by successive generations of cylindrical ducts resulting from the binary ramification of their antecedent. In its consecutive generations ($n=0,1,2,...,23$), the tree $\mathcal{T}$ begins in the trachea ($n=0$), which subdivides into two bronchi ($n=1$) for getting into the left and right lungs, respectively. Inside both lungs, the bronchi suffer more and more successive binary divisions to finally arrive in the alveoli, where the exchanges of O_2 and CO_2 take place. It is in the transport region, which corresponds to the first 16 generations, where the respiratory sound is mainly produced.

From generation to generation, the bronchi radii and lengths diminish suggesting an underlying fractal geometry. Accordingly, some works have attained to estimate the $\mathcal{T}$ fractal (Hausdorff) dimension where results varying from $D(\mathcal{T})\sim 2.60$ to $D(\mathcal{T})\sim 2.80$ were obtained [5,7]. Motivated by the fractal architecture of the lung ramification scheme, which exhibits geometrically approximate self-similarity, the present discussion is dedicated to the question whether affine spectral characteristics and self-similar structure imposed by the boundary conditions permit some sort of "reverse engineering" which leads to an air flux description implemented in a fractal discrete scheme. Such a procedure could replace in some degree the usual continuous formalisms based in spectral analysis which is in general too complicated when a complex set of boundary conditions is involved.

We suppose that the flux-tree interactions define a multiscale flux discretization where each *discrete element* works as a kernel of interaction. For example, if the respiratory sound were a consequence of the usual resonant interaction in open tubes of length L, the discrete elements would be the Fourier harmonic modes. Even though all of them can potentially interact with the n-generation bronchi, only those with length $\lambda(n)$ satisfying $\lambda(n)=2L(n)$, $n=0,1,2,...$, will resonate as a fundamental mode in each generation. The overall addition of these resonating modes would result in the respiratory sound. The present discussion is an attempt to explain the sound production with a sort of generalization of that type of interaction scheme.

It is expected that the mentioned scheme of interacting scale takes only into consideration the most relevant component of the produced sound. In fact, only the genuine solutions of the physical equations together with the boundary conditions imposed by $\mathcal{T}$ can show the nature of those discrete elements, which may be Fourier modes, turbulence vortices, solitons, or coherent structures in general. However, it will be shown that, for the purpose of a global spectral description of the high frequency of respiratory sounds, it is not essential to know what exactly is the nature of those flux elements. Although, it would be desirable to have this information for a more detailed description, we postpone the task

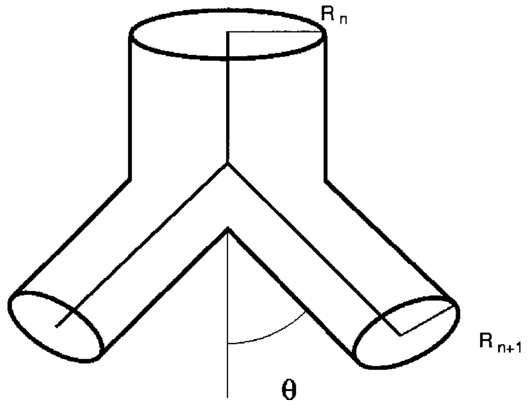

FIG. 1. Symmetric bifurcation scheme between consecutive bronchi generations of $\mathcal{T}$. Its skeleton is obtained substituting the cylinders (bronchi) of all generations by the axis segments with the same lengths. At the end, it resembles the interaction skeleton (see Fig. 2).

of finding the answer to a later time, probably via a simulation approach.

At the end, we conclude that our scheme is sufficient to explain, both qualitatively and quantitatively, the affine characteristic of the high-frequency region as a direct consequence of our hypotheses based on the fractal geometry of $\mathcal{T}$. In Sec. II we present the interaction schemes, in Sec. III the free model parameters are delimited. Discussion and conclusion are presented in Sec. IV.

II. THE INTERACTION SCHEME

We assume that $\mathcal{T}$ is a self-similar structure composed by a succession of dichotomous branchings where a parent bronchi gives rise to two smaller daughters. For simplicity, the bifurcations are taken symmetric with respect to the parent, and the bifurcation angle θ is constant throughout the generations (see Fig. 1). Consider the duct system as rigid, where variations in the overall volume of the bronchial tree may be neglected for geometric considerations.

Under a continuous approach, the transport problem to be solved must result from Navier-Stokes equations under the specific characteristics of the problem, together with the appropriate initial and boundary conditions. If $\{f^j\}$ represents the set of relevant variables (density, velocity, pressure, etc.), the deduced transport equation may have the following generic structure:

$$\mathcal{L}[\{f^j\}]=F(\{f^j\},S,\dots),\quad n=0,1,2,...,\overline{n},\tag{1}$$

where $\mathcal{L}$ is a partial differential operator over the set of variables, and the application F, besides a possible dependence on $\{f^j\}$, may include terms representing sources S or resulting from simplifications in general. For any particular physical mechanism, the interaction scheme is represented by the corresponding forms of $\mathcal{L}$ and F. If $f^j_{n,1}$ and $f^j_{n,2}$ refer to brother ducts from a $(n-1)$-generation parent, according to our hypothesis on the symmetry of bifurcation, both $\mathcal{L}$ and F should be symmetric under interchange of $f^j_{n,1}\leftrightarrow f^j_{n,2}$. From the problem solution we could then obtain the audible sound spectra.

We assume that the transport problem represented by Eq. (1) with the appropriate initial and boundary conditions can be decomposed in a sequence of multiscaled simpler problems, for each of the specific generations. Moreover, that sequence shall be invariant in scale from generation to generation. More precisely, if the solution f^j represents any of the relevant variables, and f^j_n its restriction to generation n, the scaling property

$$Z^{\lambda}f^j_{n+\lambda}=f^j_n\tag{2}$$

shall hold, where $\lambda=1,2,\dots$. In other words, the relevant dynamical conditions from generation to generation are the same if properly scaled (by scaling Z).

Instead of setting up the specific form and solving the respective dynamical equation (1), the present discussion starts from the symmetry and scaling properties of Eqs. (1) and (2) together with the phenomenological fact that resonant sound frequencies are related to the geometric dimensions (i.e., length scales) of a resonant body, as, for instance, with the pipes in a church organ or with a drum [8,9]. To be more specific, since the spectrum is intimately related to the underlying flux dynamics, Eqs. (1) and (2) suggest that the spectrum may be generated by a scheme analog to the fractal architecture of the bronchial tree. Accordingly, the fluid–bronchi interaction is represented via a nonuniform (multiscaled) discretization of the air flux into elements and a selective scheme for the interacting scale for each generation. In this case, the proposed discretization to be introduced below reflects the before-mentioned scaling properties of Eqs. (1) and (2) which is closely related to the geometry of $\mathcal{T}$.

For the case of an equally divided flux into the subsequent bronchi, the physical scheme of interactions between the flux elements and the bronchi is equivalent to the uniform Cantor set $\mathcal{C}$ of lacunarity $(N;d)$ [10,11], for an integer $N\geqslant 2$ and a real $d>N$. More precisely, we suppose that the succession of interactions throughout $\mathcal{T}$ generations follows the widely known uniform $(N;d)$ Cantor set steps of construction, in which each basic interval $I_{k,n}$ of step n is replaced by N equally spaced subintervals, the ends of $I_{k,n}$ coinciding with those of $I_{k,n+1}$, and whose length ratio satisfies $|I_{k,n+1}|=d^{-1}|I_{k,n}|$, for all n. This procedure is repeated *ad infinitum* until the uniform $(N;d)$-Cantor set is obtained with Hausdorff dimension $D(\mathcal{C})=\log_d N$. Equivalently, the interaction scheme could be represented in a tree-like topology $\mathcal{P}$ like in Fig. 2, where each segment of its nth generation represents the interaction of the air flux with the current bronchi of $\mathcal{T}$. Then, $\mathcal{P}$ represents a skeleton of interaction of the respiratory flux with the consecutive generations of $\mathcal{T}$. The fractal dimension of $\mathcal{P}$ is also given by $D(\mathcal{P})=\log_d N$ [10,11], where dichotomous branching $(N=2)$ is assumed.

As mentioned before, the interactions of the discrete elements with themselves as well as the dissipation cutoff effects are considered negligible. As the air flux passes through the bronchial tubes, part of the interacting elements kinetic energy is transformed into sound via interaction with the

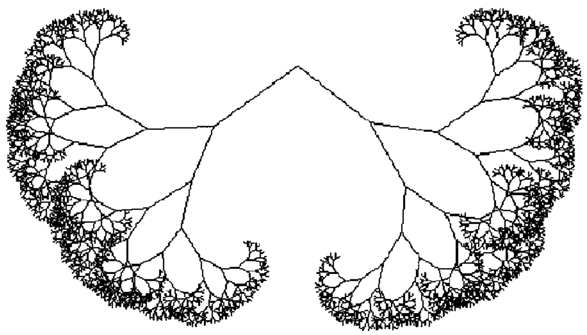

FIG. 2. Interaction tree, for $n=1,2,...,10$ (trachea is not shown), which resembles the skeleton of $\mathcal{T}$.

tubular structure. We adopt two quantitative hypothesis about the sound production due to that interaction. First, we suppose that, at each bronchi, the produced sound comes from density fluctuations with main (or pitch) frequency $\omega(n)$ proportional to some power of typical length scale $L(n)$ of the interacting bronchi,

$$\omega(n)=\kappa L^{-\gamma}(n), \tag{3}$$

where κ is a proportionality constant. The parameter γ defines the specific power law connection between sound frequencies and typical fluid length scales, which in the simplest case is $\gamma=1$. We similarly assume that the corresponding amplitude $P(n)$ is also proportional to some power of $L(n)$,

$$P(n)=\sigma M(n)L^{\delta}(n), \tag{4}$$

where σ is a proportionality constant and $M(n)$ is the number of discrete elements interacting in generation n. The referred characteristic length scales can be bronchi radius $L(n)=R(n)$ or height $L(n)=H(n)$, but both hypotheses result equivalently if we suppose that the ratio $R(n)/H(n)$ is approximately a constant, which is roughly in accordance with some geometric models for $\mathcal{T}$ ([5] and references therein).

Both hypothesis are compatible with many sound production scenarios [8,9]. For example, it is what happens when sound is produced by a harmonically bouncing body into the air. If L is a typical scale of the body, sound with frequency given by Eq. (3) with $\gamma=1$ is produced. Its intensity is proportional to the mean of the squared second time derivative of the volume of the emitting body, i.e., to $\overline{\ddot{V}^2(n)}\equiv\omega^4V^2$. The same intensity can be also written as proportional to the squared amplitude. Then, the amplitude of the produced sound is proportional to ω^2V, which is proportional to $L^{-2}L^3=L$, a power of L.

For each generation n, $M(n)$ of the above-mentioned flux elements produce sound according to two possible complementary scenarios.

(1) Sound production is associated with those $M(n)=2^{n-1}(d-2)$ annihilated fluid portions, corresponding to the discarded intervals in Cantor set construction.

(2) Sound production is associated with those $M(n)=2^n$ fluid portions allowed to pass to the next bronchi generations, which correspond to the kept intervals in Cantor sets construction, and thus could represent a nozzle-like effect.

Note that the topology of $\mathcal{P}$ for both scenarios is the same except that the construction of $\mathcal{P}$ for the second scenario is one step ahead in relation to the first.

One may employ self-similarity and scale $L(n)=d^{-n}L(0)$ back to the zeroth generation, and write $L^{\delta}(n)=d^{-\delta n}L^{\delta}(0)$. For both scenarios Eq. (4) becomes

$$P(n)=\frac{M(n)}{2^n}d^{[D(\mathcal{P})-\delta]n}P(0), \tag{5}$$

where $P(0)=\sigma L^{\delta}(0)$ is the amplitude of the produced sound in the zeroth generation and the Hausdorff dimension $D(\mathcal{P})=\log_d 2$. In Decibel scale Eq. (5) reads

$$P(n)|_{\mathrm{dB}}=20\log_{10}[P(n)/P(\bar{n})], \tag{6}$$

with $\bar{n}$ the last generation where detectable sound is produced (i.e., the highest frequency appearing in the experimental spectrum).

Under the scaling rule, Eq. (3), the equation above yields

$$P(n)|_{\mathrm{dB}}=A\log_2\frac{\omega(n)}{\omega(\bar{n})}+B, \tag{7}$$

where we have tacitly absorbed constants in the two parameters A and B, which are given by

$$A=-20\gamma^{-1}[\delta-D(\mathcal{P})]\log_{10}2 \tag{8}$$

and

$$B=-20\gamma^{-1}[\delta-D(\mathcal{P})]\log_{10}\frac{\omega(\bar{n})}{\omega(0)}, \tag{9}$$

where the frequencies have been expressed in octaves with relation to the highest detectable $\omega(\bar{n})$ frequency,

$$\omega(n)=d^{\gamma(n-\bar{n})}\omega(\bar{n}). \tag{10}$$

The result given by Eq. (7) is in accordance with the findings of [1] and [2]. It is noteworthy that independently of the scenario, where either $M(n)=2^{n-1}(d-2)$ or $M(n)=2^n$, the resulting spectra are identical, i.e., the fractal scheme implies a Babinet-like principle [12] and consequently, from the spectral shape alone, it is not possible to identify whether scenario 1 or 2, or a mixture of both, is the cause for the observed breath sound.

III. DELIMITING THE MODEL PARAMETERS

The relevant parameters for our scheme are d, δ, A, and γ. Let us adopt $\gamma=1$, which is consistent with many production sound situations like in (open or semi-open) pipes, resonators, pulsating bodies, percussion on membranes, etc. [8,9]. We now use the works of Gavrieli and coworkers [1,2] to

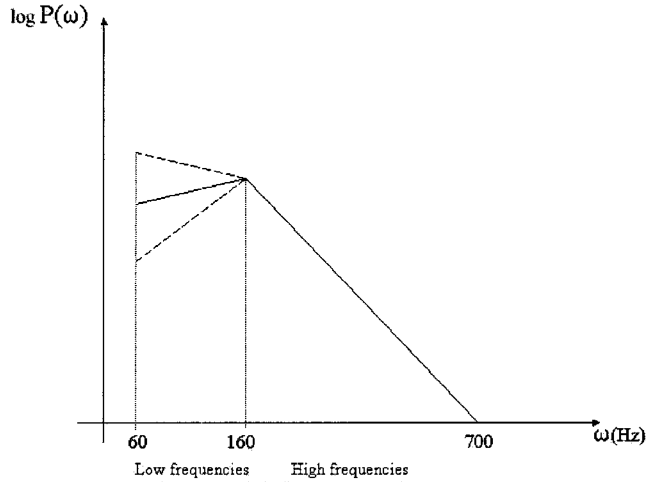

FIG. 3. Typical spectrum shape according to Gavriely *et al.* [2]. The dashed lines represent the amplitude variability usually seen in the low-frequency region. Note that the frequencies are distributed in a log scale.

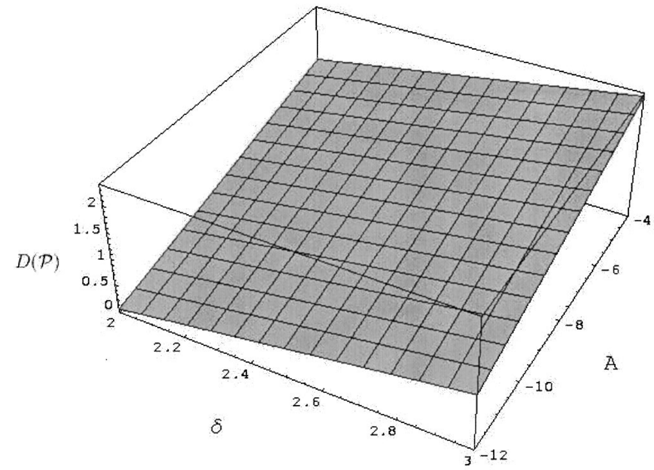

FIG. 4. Graphic of $D(\mathcal{P})$ as a function of δ and A [see Eq. (8)].

establish the limits for A, and of Nelson and Manchester [5] and of Stypa [7] to select the values for d, since $D(\mathcal{P}) = \log_d 2$.

First note that if we assume that the lowest frequency is produced in generation 4, from Eq. (10), we have

$$\overline{n} = 4 - \log_d \frac{\omega(4)}{\omega(\overline{n})}. \tag{11}$$

Let us adopt $D(\mathcal{P}) = 1.7$, as reported in [5]. According to [2] $\omega(4) \approx 150$ Hz. Then, under our hypothesis, sounds with frequencies up to about $\omega(\overline{n}) \approx 8800$ Hz are produced up to generation $\overline{n} = 13$. For the same parameter set, $\omega(\overline{n}) \approx 1200$ Hz is produced up to generation $\overline{n} = 9$. This may indicate a loss of higher frequencies due to filter properties in sound acquisition by the usual auscultation procedure.

According to [1] and [2], the amplitude spectra slopes for the high-frequency region in normal subjects when in inspiration varies between -12.7 and -15.2 dB/oct. For expiration, it varies from -13.4 to -20.3 dB/oct. However, this reflects the respiratory sound already attenuated by the physical characteristics of the chest. In fact, once generated in the bronchial tree, the pressure oscillations propagate through a sequence of tissues (parenchyma, bones, muscles, and fat) until be capted by a stethoscope. According to the literature, the transfer function of the chest cavity can be approximately supposed as log-log affine with a slope of about -8.0 dB/oct [13]. This means that the original slopes A of the auscultated sound are about -4.7 to -7.2 dB/oct for inspiration and about -5.4 to -12.3 dB/oct for expiration. Figure 3 illustrates the typical spectrum shape.

The scaling parameter d is associated to both geometries of $\mathcal{P}$ and $\mathcal{T}[D(\mathcal{T}) = D(\mathcal{P}) + 1]$ and then, it must be the same for inspiration and expiration. Since $2 \leqslant D(\mathcal{T}) \leqslant 3$ [5,7], or equivalently $1 \leqslant D(\mathcal{P}) \leqslant 2$, we have $1.25 \leqslant d \leqslant 1.41$. Figure 4 illustrates the dependence of $D(\mathcal{P})$ as a function of δ and A, obtained from Eq. (8), for the relevant ranges. The level sets for $D(\mathcal{P})$ give the possible combinations for the values of A and δ. For example, we could conclude that the most likely integer values for δ are 2 and 3 for inspiration and expiration, respectively. This would indicate that the physical mechanisms of respiratory sound production in inspiration and expiration have fundamental differences: while in the first case the amplitudes are proportional to $L^2(n)$, in expiration the proportionality is $L^3(n)$.

The low-frequency spectral region (75–150 Hz) does not fit an affine rule as precise as usually seen in the high-frequency region. Moreover, even for the same subject, it can vary substantially, depending on the auscultation point. However, a significant amount of the generated sound may be due to equivalent mechanisms as the ones covered by our schemes. In fact, the reported values for the angular coefficient A range over values usually larger than those found for the high-frequency region [1,2]. From Eq. (8), we can deduce that the associated fractal dimension is higher for the first tree generations than the others or, equivalently, the associated d values are smaller, which is in agreement with the Weibels model [14,6] and with those who think that a multifractal model is more appropriate to represent $\mathcal{T}$.

IV. DISCUSSION AND CONCLUSION

The proposed model for high-frequency respiratory sound generation is based on a discrete scheme of interaction of the flow with the usually observed bronchial tree fractal geometry, in which the air flux interacts with $\mathcal{T}$ according to the topology of a Cantor based tree (skeleton) $\mathcal{P}$. We consider two complementary scenarios of interaction where either the stopped or the passed fluid portions are responsible for sound production. In both cases, the flux kinetic energy is converted into sound with both intensity and frequency proportional to powers of the discrete element producing sound. In both scenarios the interaction skeleton $\mathcal{P}$ is the same, with dimension given by $\log_d 2$. The model lead us to conclude that, for both scenarios, the log-log spectra concerning to the high-frequency region are affine with slope A determined by $\mathcal{P}$'s fractal dimension end the relation given by Eq. (8). That relation is consistent with some estimations for both A and $D(\mathcal{T})$ [1,2,5,7,15].

The consistency of predictions with the observations suggests that the connection between the actual physical mechanisms given by Eq. (1) and the boundary conditions defined by $\mathcal{T}$ should fit the proposed schemes. Therefore, even not being sufficient to completely determine the physics of the respiratory sound production, some insight can be taken from the present study. Moreover, the present discussion is in accordance with the simulations carried out by Almeida *et al.* [16] of Navier-Stokes equations in two-dimensional ramified structures like $\mathcal{T}$. These authors found strong indication that the flux distributions along $\mathcal{T}$'s generations are set up under a self-similar scheme in which a binary model was proposed. If the analogy of our hypothesis on the symbolic equation (1) with those indications where perfect, the two-dimensional Navier-Stokes equations numerically integrated in [16] could be further simplified to fit the format of that symbolic transport equation and its solutions would satisfy Eq. (2).

Under the supposition that the geometries of $\mathcal{P}$ and $\mathcal{T}$ are equivalent, we must conclude from the estimations for A and $D(\mathcal{T})$ that the physical mechanisms for sound production in inspiration and expiration are not the same. It is important to note, however, that the hypothesis of equivalence between $\mathcal{P}$ and $\mathcal{T}$ geometries may be relaxed, and are associated to different scaling parameters d_1 and d_2, respectively. This gets our schemes richer in possible scenarios, but still satisfying the affine high-frequency spectral characteristics. For example, if we suppose that the physical mechanisms for inspiration and expiration are the same, we must conclude that the dimension of $\mathcal{P}$ depends on the phenomena (inspiration or expiration), i.e., the flux energy partition is different for each of them. In this case, the geometries of $\mathcal{P}$ and $\mathcal{T}$ are at most related, but not the same. The value $\gamma=1$ assumed to discuss some numerical results in the previous section could also be relaxed.

Besides the consistency of the predictions with the observations, the discrete approach adopted in our model can find some formal justification by considering that some turbulent regimes have been treated as resultant of a number of uncoupled coherent structures like solitons [17]. These formalisms have been successful in reproducing the overall observed spectral characteristics of such regimes. Accordingly, in the proposed model, only weak nonlinear interactions between such structures are implicitly taken into consideration by the discretization scheme. Therefore, if the consistency of our model with respect to observations reflects physical equivalence, one may conclude that such coupling effects are not relevant in the high-frequency lung sound production. Further, recent evidence of chaotic regimes in the overall respiratory sound frequency [18] might be traced back to the low-frequency spectral region, which corresponds to the first bronchi generations ($n \leqslant 4$). In fact the pressure perturbations in that region may be sufficiently intense for those coupling effects to produce nonintegrable chaotic regimes. However, such perturbations are out of the validity region of our model, a topic to be considered in a future work.

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